Lasing Interactions Disclose Hidden Modes of Necklace States in the Anderson Localized Regime

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ABSTRACT: Anderson light localization is known to be minimally affected by nonlinearity (e.g., gain), as a lasing mode typically arises from an isolated localized mode in a passive system. However, in the Anderson localized regime of low-dimensional systems, local resonances can occasionally experience strong level overlap and form an asymmetric single line, which is also known as necklace states. This case is not a hypothetically rare event and indeed is not uncommon. For such passive transport, we experimentally and theoretically investigate how nonlinearity affects necklace states in multilayered systems. When gain or amplifying media are introduced in a system, an asymmetric line splits into multiple lasing peaks, interacting with one another. By decomposing field spectra into a sum of Lorentzian lines, we find that diverging lasing peaks arise from quasimodes; in turn the lifetime and degree of level overlap of the underlying quasimodes are strongly attributed to the threshold and spectral behavior of the emerging lasing modes. These results suggest that observation of lasing interactions could be an alternative yet powerful tool for investigating necklace states (or hybridized states of coupled resonances), the existence of which is challenging to experimentally prove and study, due to grossly single-appearing spectral peaks in passive systems.

KEYWORDS: disordered photonics, random lasers, coupled resonances, quasimodes, low-dimensional systems, transfer matrix method, nacre

In general, lasing modes are considered to represent a subgroup of eigenmodes excited in the localized system. A few dominating modes deplete the gain of excited molecules (i.e., gain competition) when the frequencies of the neighboring eigenmodes are very close to one another. In a diffusive regime, multimode lasers exhibit strong modal interactions in the presence of nonlinearity (i.e., gain). In the Anderson localized regime, interference-mediated passive transport without gain can be understood by the degree of level overlap, which is also known as the Thouless number $\delta$ for a fundamental localization parameter. Anderson localization supports strong wave confinement with a high quality (Q) factor. Localized modes are sharply divided or isolated in the spectrum with the average spectral space $\Delta \nu$: $\delta \equiv \Delta \nu / \Delta \nu < 1$. In this case, a localized mode provides such a sharp resonance to induce lasing in gain. Because localized modes are minimally overlapped in frequency and space, modal interactions are known to be suppressed such that a lasing mode grows from a single localized mode of the passive cavity. When $\delta < 1$ in low-dimensional systems (e.g., 2D and 1D), local resonances can occasionally be hybridized in space and frequency and form necklace states, which have a long chain of multiple field peaks in the sample. In frequency, necklace states are manifested in the form of a single asymmetric passive line, resulting from strong coupling of spectrally adjacent multiple lines. Importantly, such strong coupling is not a hypothetically rare event. In 1D systems, necklace states have been the object of theoretical investigation as well as experimental studies using disordered multilayered nanostructures found in natural and synthetic systems.

Even in 2D photonic systems, hybridization of local resonances was successfully studied. Importantly, necklace states are known to play an outsized role in passive transport. In the presence of gain, relatively little is, however, known for amplification and lasing of necklace states. Due to the coupled nature of multiple resonances, amplified modes in necklace states could potentially affect one another. Obviously, such a case is contradictory to the notion that Anderson localization suppresses interactions in lasing modes. Thus, it is particularly of interest to investigate lasing interactions within a grossly single-appearing spectral peak of necklace states in passive systems.

For hybridized states of coupled resonances in passive systems, the field intensity spectra can be expressed by a...
superposition of decaying Lorentzian components, which are
defined as quasimodes, at each location within a sample.23,24
The question then arises as to what happens if the underlying
quasimodes in necklace states are amplified in the presence of
gain. Does one lasing mode arise from the grossly single-
appearing peak? Or does one lasing mode correspond to one of
the decomposed quasimodes? If so, can the lifetime and degree
of level overlap of the underlying quasimodes be correlated
with the threshold and spectral behavior of the emerging lasing
modes? These questions can be addressed by a modal
decomposition analysis, which enables (i) full comparison of
the spectral properties of quasimodes (i.e., linewidth and
central frequency) with those of lasing modes and (ii)
quantification of overlap between quasimodes that leads to
spectral repulsion toward the level spacing equilibrium at high
excitation energy.

In this Article, we explore interactions of lasing that arise
from necklace states in the Anderson localized regime. First, we
experimentally demonstrate that spectral variations of lasing
modes (e.g., splitting and repulsion) are not uncommon in low-
dimensional localized samples. We also characterize the angular
and polarization properties of lasing modes and their coupled
states. Second, we numerically simulate experimentally
observed dynamic responses of lasing modes by introducing
the imaginary part of the refractive index of gain media to
excitation energy-induced polarization. Third, we computa-
tionally extract quasimodes in necklace states by decomposing field
spectra into a sum of Lorentzian lines, in which the central
frequency and the linewidth are independent from the sample
location. This modal decomposition analysis allows us to
quantify the lifetime and degree of overlap of underlying
quasimodes. As spectral overlap between quasimodes increases,
the central frequency of lasing modes is largely shifted, leading
to mode repulsion. Moreover, the lifetime of quasimodes
obtained from a time–frequency analysis is shown to be
strongly correlated with the threshold of emerging lasing
modes. Overall, experimental observation and theoretical
prediction of lasing properties allow us to better understand
the characteristics of necklace states (or coupled resonances),
which otherwise would be intrinsically hidden under a grossly
single-appearing asymmetric line in a passive system.
Since interference-mediated transport for both classical and
nonlinear waves was a challenging theoretical and experimental
problem, multilayered structures have widely been
used.6,8,10−13,16,18,20−23,25,26 Nature has many different types
of layered nanostructures. For example, nacre (mother-of-
pearl) is an immediately exploitable localized sample.16 The
highly multilayered nanostructure of nacre found in nature is
beyond synthesized structures using conventional nanofabrica-
tion methods. Nacre is composed of microscopic crystals of

Figure 1. Natural Anderson-localized samples of nacre extracted from abalone shells. (a) Schematic illustration and scanning electron micrograph
showing nearly parallel multilayered nanostructures of nacre (mother-of-pearl). (b) Photograph of sliced nacre after removing protein pigments. (c)
Photograph of abalone seashells. (d) Schematic illustration of photoluminescent measurements. To excite localized modes in nacre, linearly polarized
short pulsed illumination is incident normally on the multilayered nacre structure. The excitation energy is controlled with a linear variable neutral
density filter. The lasing emission Ie is characterized in multiple domains of angle, polarization, and spectrum.
calcium carbonate, primarily aragonite (Figure 1a). Specifically, nacre has favorable features for shortening the localization length of light and enhancing the phase randomization as follows: (i) The thickness of the aragonite layer ($d_i$) is comparable to the wavelength of light, $\lambda \approx 500$ nm. (ii) The variation in $d_i$ is high with a standard deviation of $\sim 10\%$. (iii) Nacre is composed of a large number of aragonite layers ($\sim 4000$ layers in a 2 mm nacre sample). (iv) The low structural dimension (i.e., quasi-1D) restricts the volume explored by light waves. Indeed, nacre has inspired a broad range of synthesis and fabrication nanomaterials to mimic its intriguing optical properties. 27

Figure 2. Experimental detection and numerical simulation of lasing mode interactions. (a) Experimental observation of lasing mode variations as the excitation energy $E_{ex}$ increases. The emission spectra are vertically shifted based on a scale of $E_{ex}$. Inset: Chemical structure of DCM. (b) Measured emission intensity $I_e$ as a function of $E_{ex}$. (c) Spectral positions of mode 1 and mode 2 marked in (a) and their difference as a function of $E_{ex}$. (d) Measured emission spectra at different angular directions. The exact forward direction is 0°. Inset: Dependence of the emission intensity on the detection angle. (e) Measured co-polarized and cross-polarized emission spectra at two different locations. The green lines indicate necklace states. (f) Simulation of lasing mode interactions as the density of excited atoms $N_{ex}$ increases. (g) Computed emission intensity $I_e$ and linewidth of mode B and mode C marked in (f) as a function of $N_{ex}$. (h) Spectral positions of mode B and mode C and their difference as a function of $N_{ex}$.
We experimentally show lasing mode splitting and repulsion from a low-dimensional localized sample (i.e., highly multilayered nanostructures of nacre). A nacre sample is sliced from an abalone shell using a tile saw. To effectively introduce gain, we remove protein pigments that fill the gap between aragonite layers using a sodium hypochlorite (NaOCl) solution (6%). The deproteinization changes the captivating optical reflection of an abalone shell into a bright white color. Then, the deproteinized nacre sample is doped with rhodamine 6G with a concentration of 0.5 mg/mL in methanol or DCM. Then, the deproteinized nacre sample is doped with rhodamine 6G with a concentration of 0.5 mg/mL in dimethyl sulfoxide (DMSO). In particular, DCM in dimethyl sulfoxide (DMSO) is useful for obtaining stable lasing emission over a relatively long time (≈5 min) to capture the exact dynamics of modes while minimizing any signal artifact and uncertainty. To excite localized modes in nacre, a linearly polarized excitation beam from a frequency-doubled Q-switched Nd:YAG laser (pulse repetition rate = 100 Hz, pulse duration = 400 ps, and λ = 532 nm) is incident normally on a 300 μm2 sample area with a low numerical aperture objective (5×). The excitation beam is spatially fixed on the sample surface, while being focused into a spot with a full width at half-maximum (FWHM) of ≈8 μm, avoiding spectral line broadening. In the detection arm (Figure 1d), a polarizer selects the polarization states of lasing emission so that the co-polarized component and the cross-polarized component of the emitted light are recorded independently. In addition, angle-resolved measurements of lasing emission are conducted using a goniometer coupled to a spectrometer with a resolution of ≈0.2 nm.

Figure 2a shows lasing spectra from the nacre sample with a thickness of 2 mm. The emission intensity of spectra has clear threshold behavior (Figure 2b). At the lasing threshold, a lasing peak emerges. As the excitation energy further increases, we observe dynamic responses of lasing frequencies (Figure 2c), while generating bimodal lasing peaks from a single lasing peak (i.e., mode splitting) (marked with mode 1 and mode 2 in Figure 2a). In Figure 2d, the lasing emission is highly directional, as nacre is a multilayered nanostructure that restricts the volume explored by light waves into the normal direction of layers. When the input excitation beam is incident normally onto the surface, the effective excitation/amplification of localized modes in nacre can be achieved. The transmission and the reflection in nacre closely follow light transport in 1D, the output lasing emission should be highly directional with respect to the incident beam. Thus, angular deviations from the major propagating detection (perpendicular to the layered structures) merely impose a constant prefactor in the emission spectra, while freezing the spectral positions of modes. We note that angle-resolved measurements in disordered systems are not ideal to systematically control or study coupling regimes of modes in a broad spectral range and that temperature tuning or postfabrication methods are often used to modify the dielectric environment of disordered media.

In addition, we study the effect of light amplification on polarization in the Anderson localized regime. In Figure 2e, almost all lasing modes are not polarized even with the linearly polarized excitation. Random lasing modes in the strongly scattering regime are known to not be polarization-sensitive, because strong scattering in dye molecules randomizes or scrambles the polarization. On the other hand, we find some bimodal lasing peaks from necklace states are relatively polarization-sensitive. Overall, although the lasing interactions and their polarization states have easily detectable spectral features, it is difficult to resolve the exact spectral profile of necklace states in passive systems due to the limited spectral resolution.

To understand the origin of the diverging lasing peaks and their spectral variations as a function of the excitation energy, we numerically investigate the evolution of optical waves with gain in 1D systems, using the time-independent transfer matrix theory (Methods). Mimicking the experimental conditions, we model nanostructures that have 2000 dielectric...
layers with alternating high-index layers $n_a \approx 1.65 - 1.68$ and lower index gap layers $n_g$. The thicknesses of aragonite $d_a$ and gap $d_g$ layers are set to be in the ranges of 473 ± 50 nm and 40 ± 10 nm, respectively. Such a disordered system results in a localization length of $\sim 37.2 \mu m$. This short localization length ($\ll$ total system length of 0.5 mm) ensures that the waves are deeply localized. The gain medium (i.e., rhodamine 6G in methanol) in the gap layers is simulated by adding excitation energy-induced polarization $\chi_g(N_{ex}, \nu)$. Figure 2f shows the computed field intensity $I(=E\cdot E^*)$ at the sample output surface as a function of excitation energy $N_{ex}$. In the absence of gain (Figure 3a), the passive transmission $T$ has two narrow peaks of isolated localized modes at $\nu = 541.69$ and 541.92 THz and one relatively broad asymmetric single passive line at $\nu = 541.83$ THz. When $N_{ex} = 0.016$ (Figure 3b), the single asymmetric line split into two narrow emission peaks $I_e$ and the linewidths of the emerged peaks become narrower than their spectral separation of $\sim 0.02$ THz (Figure 2g). Accounting for the central frequencies of the peaks (Figure 2h), the emerged lasing peaks (blue and purple lines) are largely shifted, compared with the spectrally fixed isolated mode (gray line). As the excitation energy increases, the necklace states show mode competition and repulsion, leading to spectral separation between selected bimodal lasing peaks, while the spectral positions of the isolated lasing modes are frozen. An increase in the excitation energy in the presence of gain (i.e., amplifying medium) can be understood as an increase in the oscillator strength (or $Q$ factor) promoting mode splitting in frequency; the characteristics of optical resonances in disordered media resemble the conventional oscillator. Indeed, this prediction (Figure 2h) is in good agreement with the experimental result of mode splitting (Figure 2c).

From a phase shift of waves, we find a clue for the existence of hybridized/coupled states inside the asymmetric peak. At $\nu = 541.69$ and 541.92 THz in Figure 3a, the transmission of the narrow peaks experiences an accumulated phase shift of $\pi$, which is a hallmark of typical isolated localized modes. However, as far as the asymmetric single line is considered, the
phase undergoes a 2π shift between \( \nu = 541.8 \) and 541.9 THz. Importantly, this indicates that the transport in the asymmetric single passive line is mediated by hybridized states of two possible hidden quasimodes, of which the central frequencies are closely clustered.

To explicitly demonstrate the presence of hybridized states within the asymmetric peak, we conduct a modal decomposition analysis. The field spectrum \( E \) at each location \( z \) inside the sample is fitted by a sum of Lorentzian lines:\n
\[
E_{\nu}(\nu, z) = \sum_{n=1}^{N} E_n(\nu, z) = \sum_{n=1}^{N} \frac{A_n(z)}{\Gamma_n(z) + i(\nu - \nu_n(z))}
\]

where \( \Gamma_n \) is the linewidth, \( \nu_n \) is the central frequency of the resonance, \( A_n(z) \) is a complex coefficient, and \( N \) is the number of quasimodes. \( N \) is chosen to be 5 to include isolated modes and slowly varying background. \( A_n(z), \Gamma_n(z), \) and \( \nu_n(z) \) are inversely found to minimize the objective function \( F \):

\[
\min F = \sqrt{\frac{1}{S} \sum_{i=1}^{S} \left\| E(\nu_i, z) - \sum_{n=1}^{N} \frac{A_n(x)}{\Gamma_n(z) + i(\nu_i - \nu_n(z))} \right\|^2}
\]

where \( \nu_i \) is the sampling frequency at which \( F \) is evaluated, \( \| \ldots \| \) is the Euclidian norm, and \( S \) is the number of \( \nu_i \) within the frequency of interest. To escape from local minima, we combine stochastic (i.e., genetic algorithm) and deterministic (i.e., Nelder–Mead simplex) optimization methods that use multiple groups of initial guesses.

In Figure 4a, the field spectra of the passive system in the hybridized state include relatively broad spectral peaks inside the sample; the vertical ridges trace the spectral maxima of \( I(\nu, z) \). Figure 4b depicts the Lorentzian terms in eq 1, of which the summation fits to the field spectra of Figure 4a at the different locations \( z \) inside the sample. As \( \Gamma_n(z) \) and \( \nu_n(z) \) are specified, each Lorentzian term corresponds to a quasimode. In the range between \( \nu = 541.8 \) and 541.9 THz, the modal decomposition shows that the two quasimodes (i.e., modes B and C in Figure 4b), the linewidths of which are larger than their spectral separation, mainly contribute to the single asymmetric peak of the hybridized state. We note that mode E is simply the slowly varying background. As shown in Figure 4c, this decomposition is valid for the entire locations \( z \) inside the sample. Importantly, the summation of decomposed quasimodes \( E_{\nu} \) is nearly identical to \( E \) (Figure 4d). This result supports the idea that the decomposed quasimodes (Figure 4c) are a complete set of hidden resonances in the asymmetric transmission spectrum. Thus, the set of the hidden quasimodes could potentially mediate the splitting of lasing modes shown in Figure 2f.

In this case, we investigate how the lasing mode behavior is associated with the underlying passive quasimodes. The modal decomposition allows us to characterize a spectral shift \( \Delta \nu_{\nu} \) between a quasimode and a lasing mode in the hybridized state. Each Lorentzian term in eq 1 is defined by the central frequency \( \nu_{\nu} \) which is independent over locations with random fluctuations (Figure 5b). When gain is introduced, the fluctuation in \( \nu_{\nu} \) disappears, which in turn sharply defines the lasing mode in the field spectra (Figure 5a). In Figure 5b, the spectral spacing between the lasing modes is larger than that of the quasimodes, resulting in \( \Delta \nu_{\nu} \) toward the level spacing equilibrium. The quasimodes behave like typical eigenmodes; the quasimodes overlapped in frequency undergo gain competition (i.e., gain depletion) and then repel one another, increasing the frequency spacing between the adjacent lasing modes.

Figure 5. Interplay between spectral overlap of quasimodes and spectral variations of lasing modes. (a) Spatial field spectra \( I(\nu, z) \) of the lasing system. The lasing modes, mode B (left) and mode C (right), are excited with different \( N_{\nu} \). (b) Central frequencies \( \nu_n \) of mode B and mode C over locations \( z \) in the passive and lasing systems. (c) Borderlines of the spectral extent of quasimodes, with modes A–D demarcated using an intensity cutoff \( I = 0.2 \). (d) Spectral shifts of modes \( \Delta \nu_n \) as a function of spectral overlap between quasimodes \( O_{\nu} \).
To capture the degree of overlapping, we delineate the spectral extent of each quasimode, using an intensity cutoff of ~0.2 (Figure 5c). The borderlines clearly visualize the spectral overlap between the field spectra of the adjacent quasimodes (modes B, C, and D). The spectral overlap varies over z. At the output surface (z = 0.52 mm), the spectral overlap is insignificant. Inside the sample, the spectral overlap is significant through the exponentially falling tails (Figure 5c and c). To quantify the exact degree of overlapping, we calculate an overlapped spectral area of the quasimode $O_n$ with the neighboring quasimodes:

$$O_n = \sum_{m \neq n} \int_{0}^{\infty} \int_{-\infty}^{\infty} \min\{I_n(\nu), I_m(\nu)\} d\nu dz,$$

where $I_n = E_n E_n^*$ is the field intensity of decomposed modes, $I_n$ is the adjacent quasimodes of $I_n$ and $z_0$ is the location of the output surface. As a result, Figure 5d shows the relationship between $O_n$ and $\Delta \nu$. For example, mode C in Figure 5c has the largest calculated overlapping area and the largest $\Delta \nu$. This result explains that spectral repulsion becomes stronger as the adjacent quasimodes are closely overlapped. In this nominally localized sample with $\delta < 1$, the interactions of lasing modes increase due to the spectral overlapping of the underlying quasimodes. Vice versa, this observation of lasing interactions can be used to qualitatively understand the coupling of quasimodes, which is intrinsically hidden under the asymmetric line of necklace states.

It is also important to predict a mode that will lase first within the overlapped spectrum. For this prediction, we study the lifetime of quasimodes using a time–frequency analysis. In Figure 6a, the decomposed quasimodes at $\nu = 541.82$ and 541.84 THz (i.e., modes B and C) have different linewidths $\Gamma_n$ of which the values of which are relatively consistent over the location $z$ (Figure 6b). By transforming each decomposed quasimode spectrum $E_n$ into the time domain, we obtain a temporal response of each quasimode, which is a pulsed transmission. The pulsed transmission is the Fourier transform of the product of the quasimode spectrum $E_n$ in eq 1 and the Fourier transformed Gaussian pulse $E_0$:

$$E_0(\nu) \approx \exp\left(-\frac{(\nu - \nu_0)^2}{2\sigma^2}\right)$$

with $\sigma = \frac{1}{2\pi \sigma_\nu}$

where $\nu_0$ is the carrier frequency and $\sigma_\nu$ is the Gaussian temporal pulse width. In the time domain, the incident Gaussian pulse is

$$E_0(t, \nu_0) \approx \exp\left(-\frac{t^2}{2\sigma^2}\right)\cos(2\pi \nu_0 t)$$

In Figure 6c, the temporal responses are computed from the spectra in Figure 6a with an incident Gaussian pulse of $\sigma = 2.5$ ps. The decay rates of modes B and C are 16.68 and 8.98 ps$^{-1}$, which correspond to a $\Gamma_n$ of 10.7 and 22.2 GHz, respectively. The energy decay of mode B is slower than that of mode C. This leads mode B to first reach the threshold for lasing in the hybridized state (Figure 2f). Thus, the time–frequency analysis reveals the important role of the lifetime in predicting the underlying quasimodes that are likely to lase. Conversely, the order of the lasing onset provides the information on the lifetime of the underlying quasimode in the passive system.

In conclusion, we report that experimental investigation of lasing interactions can be a useful tool for better understanding Anderson light localization with necklace states in multilayered systems. Spectral profile of necklace states in passive systems is often difficult to resolve and distinguish with isolated localized modes, due to the limited spectral resolution of conventional spectrometers. On the other hand, gain amplifies quasimodes and reveals their characteristics as readily detectable spectral signals of lasing interactions. As lasing modes emerge and grow, multiple quasimodes are decoupled spectrally, which are intrinsically hidden under an asymmetric line of necklace states in a passive system. Through dynamic features in the $\nu$–$N_\alpha$ domain, we suggest an indirect yet simple approach for experimentally characterizing optical necklace states in Anderson localized systems, without relying on ultrahigh-resolution spectroscopy or time-resolved measurements.

**METHODS**

**Quasimode and Lasing Mode Simulation by Combining Transfer Matrix Method and Semiclassical Laser Theory.** To compute quasimodes and lasing modes, we use the transfer matrix method combined with the semiclassical laser theory. EM waves (E and H) of a wavenumber $k(=2\pi/\lambda)$ propagating through $L_n$ layers at the normal incidence are described such that

$$\begin{bmatrix} E_l \\ H_l \end{bmatrix} = M \begin{bmatrix} E_{l+1} \\ H_{l+1} \end{bmatrix} = M_1 M_2 \cdots M_N \begin{bmatrix} E_{l+N} \\ H_{l+N} \end{bmatrix}$$

and $M_l = \begin{bmatrix} \cos(k d_n) & i \sin(k d_n) / n_l \\ i \sin(k d_n) n_l & \cos(k d_n) \end{bmatrix}$
where \( n_i \) and \( d_i \) are the refractive index and thickness of the \( i \)th layer, respectively. \( d_s \) and \( d_t \) fluctuate around 485 and 40 nm, following normal distributions with dispersions \( s_s \) and \( s_t \), respectively. Linear gain is implemented by a spatially modulated frequency-dependent susceptibility \( \chi_g(z, k) \) in the refractive index of gap area \( n_g \):

\[
\chi_g(z, k) = \frac{A_m N_{ea}(z)}{k_a^2 - k^2 - i k \Delta k_a} \tag{7}
\]

and

\[
n_{ea}(z, k) = n_z(z, k) + i n_e(z, k) = \sqrt{\frac{1}{n_{m(\omega)}^2(z, k) + \chi_g(z, k)}}
\]

where \( n_z \) and \( n_e \) are the refractive indices of the background material, \( z \) is the spatial coordinate, \( A_m \) is a material-dependent constant, \( N_{ea} \) is the density of excited atoms, \( k_a \) is the atomic transition frequency (i.e., peak frequency of dye molecules), and \( \Delta k_a \) is the spectral linewidth of the atomic resonance (i.e., spectral width of spontaneous emission of dye molecules). To model rhodamine 6G, \( k_a = 11.38 \text{ mm}^{-1} \) (551 nm) and \( \Delta k_a = 0.72 \text{ mm}^{-1} \) (35 nm) are used. Both \( \text{Re}(\chi_g(k)) \) and \( \text{Im}(\chi_g(k)) \) are proportional to the density of excited atoms \( N_{ea} \) determined by the excitation energy. After implementing \( n_{ea}(z, k) \) in the characteristic matrix \( M \), the transmission \( T \) and the reflectance \( R \) can be obtained as follows:

\[
T = \frac{2n_0}{n_0 M_{11} + n_0 n_1 M_{12} + M_{21} + n_1 M_{22}}
\]

\[
R = \frac{n_0 M_{11} + n_0 n_2 M_{12} - M_{21} - n_2 M_{22}}{n_0 M_{11} + n_0 n_2 M_{12} + M_{21} + n_2 M_{22}} \tag{8}
\]

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